the measurement environment,
the dielectric constants,
the losses in the electrodes,
CMC microstrip effects.

Future effort will be devoted to incorporating frequency dependent loss mechanisms in the electrodes and dielectric layers. Companion publications dealing with computational aspects of the capacitor model, characterization and analytic modelling are in preparation.

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Unified Dispersion Model for Multilayer Microstrip Line

A. K. Verma and G. Hassani Sadr

***Abstract*—A unified dispersion model is presented to calculate frequency dependent dielectric constant for a multilayer microstrip line. The model is a combination of TTL method, the method for the reduction of multilayer structures to an equivalent single layer microstrip line and the Kirschning and Jansen dispersion model. The result of the model has been confirmed within an accuracy of 1% against the results from SDA, ESDT and MM i.e., various forms of full wave analysis. These results have been confirmed between 2 GHz and 18 GHz. The present model is suitable for use in a CAD package for MIC, MMIC, and printed antenna design.**

I. INTRODUCTION

The microstrip transmission line is dispersive in nature. Many rigorous theoretical formulations have been reported in the literature to obtain $\epsilon_{\text{eff}}(f)$ of the microstrip line [1]–[3]. However, the published numerical results even on the same structure, show wide variations. The main cause of the variation in results is the tendency to reduce computer time by taking a small number of basis functions. Thus, an accurate full-wave field analysis is suitable for scientific investigation and generation of a data bank, but is not suitable for use in the CAD directly. Also, computation based on a full-wave analysis with the time saving mentioned above may not be accurate.

The closed form dispersion models for the microstrip line have been proposed by various researchers for MIC and MMIC CAD. Recently, the Kirschning and Jansen (KJ) dispersion model [4] has been found to be the most accurate compared against the measured values [5]. However, none of the researchers has reported on the dependence of the overall accuracy of the KJ dispersion model and other dispersion models on the calculation of $\epsilon_{\text{eff}}(0)$ by various static methods. Moreover, our careful analysis of experimental results of Edwards and Owens [6] using the KJ dispersion model shows a degradation of results for very narrow lines. In the first part of this paper we have compared various static methods to calculate $\epsilon_{\text{eff}}(f = 0)$ with the aim of improving the accuracy of the KJ dispersion model. In the second part of the paper, we have extended the KJ dispersion model to the multilayer microstrip line. The proposed "unified dispersion model" has been used to analyze $\epsilon_{\text{eff}}(f)$ for a double layer substrate on GaAs, shielded microstrip line and covered microstrip line. Results of the proposed model have shown good agreement with calculations carried out using SDA [1], ESDT [2] and the method of moments [3].

II. COMPARISON OF STATIC METHODS FOR DETERMINATION OF $\epsilon_{\text{eff}}(0)$

Kirschning and Jansen [4] have adopted the following mathematical structure for calculating the frequency dependent effective dielectric constant,

$$\epsilon_{\text{eff}}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{\text{eff}}(f = 0)}{1 + P(f)}. \tag{1}$$

The accuracy of the dispersion expression (1) depends upon the accuracy of the determination of $\epsilon_{\text{eff}}(0)$. The static effective dielectric constant can be determined by methods developed by Bryant and Weiss [7], Hammerstad and Jensen [8], Wheeler [9] and Yamashita and Mitra [10]. Edwards and Owens [6] have carefully measured, between 2 GHz and 18 GHz the dispersion of microstrip lines on a sapphire substrate [$\epsilon_{r\perp} = 9.4$, $\epsilon_{r\parallel} = 11.6$, $h = 0.5 \pm 0.05$ mm] having w/h ratio between 0.1 and 9.14. For the purpose of comparison of $\epsilon_{\text{eff}}(0)$ calculated by four methods, the experimental values of ϵ_r and $\epsilon_{\text{eff}}(0)$ have been obtained from Edwards and Owens. The absolute value of percentage deviation of the calculated $\epsilon_{\text{eff}}(0)$ from the measured $\epsilon_{\text{eff}}(0)$ is defined as

$$K = \left| \frac{\epsilon_{\text{eff}}(0) \text{ exp} - \epsilon_{\text{eff}}(0) \text{ cal}}{\epsilon_{\text{eff}}(0) \text{ exp}} \right| \times 100. \tag{2}$$

Fig. 2 shows a significant variation in the percentage deviation or percentage error for four methods. The rms deviation of $\epsilon_{\text{eff}}(0)$ obtained from the methods of Bryant and Weiss, Hammerstad and Jensen, Variational and Wheeler are 0.13%, 0.46%, 0.08% and 0.16% respectively. Thus, Bryant and Weiss and variational methods provide the best results. The method of Hammerstad and Jensen results in deviations as high as 3% for narrow lines. The variational method gives a higher deviation for a wider strip con-

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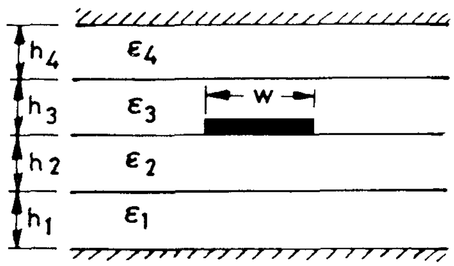

Fig. 1. Multilayer microstrip line.

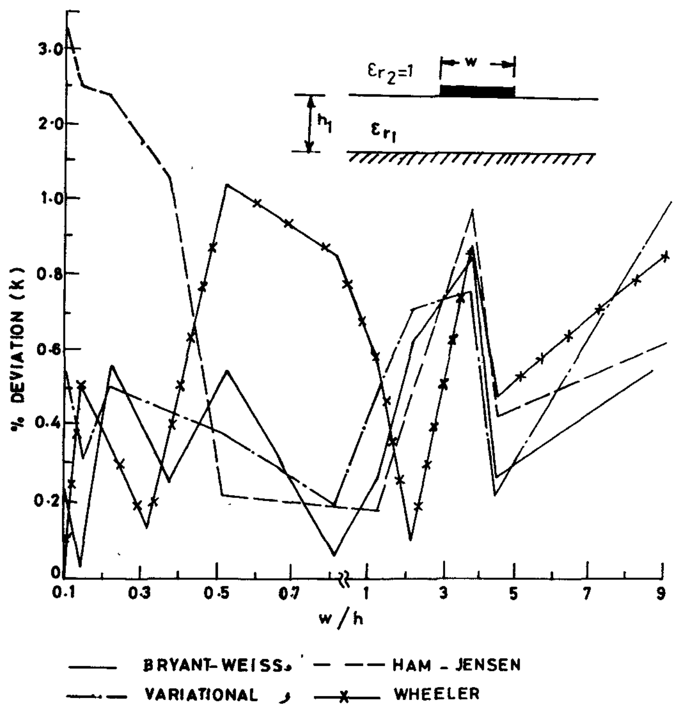

Fig. 2. % Deviation of $\epsilon_{eff}(0)$ compared to experimental $\epsilon_{eff}(0)$. (—) Bryant-Weiss, (---) Ham-Jensen, (-·-) Variational, (-×-) Wheeler.

ductor. Using the experimental values of ϵ_r from Edwards and Ownes, calculated value of $\epsilon_{eff}(0)$ by the variational method and the KJ dispersion model $\epsilon_{eff}(f)$ has been calculated to confirm the validity of the unified dispersion for the microstrip line. The calculated $\epsilon_{eff}(f)$ by the present model follows experimental results more closely than the calculated $\epsilon_{eff}(f)$ by the SDA [6]. The rms value of deviation is within 1% with significant improvement in results for the narrow lines.

III. Unified Dispersion Model

The unified dispersion model is a combination of the transverse transmission line technique (TTL) for the calculation of $\epsilon_{eff}(0)$ of shielded and multilayered microstrip lines [11], the method for the reduction of a multilayer microstrip structure into an equivalent single layer microstrip structure [12] and original KJ dispersion model for the single substrate microstrip line [4]. Instead of the KJ dispersion model any other closed form dispersion model can be used with this unified dispersion model. We shall calculate dispersion behavior of the various structures obtained from the structure shown in Fig. 1. The $\epsilon_{eff}(0)$ of this structure is given by

$$\epsilon_{eff}(0) = \frac{C}{C_o} \tag{3}$$

The variational expression for C and C_o, i.e., the capacitance per unit length of the line with dielectrics and when all dielectrics replaced by air respectively, is given by

$$\frac{1}{C} = \frac{1}{\pi\epsilon_0} \int_0^{\infty} \frac{[f(\beta)/Q]^2}{\beta} \frac{1}{Y} d\beta \tag{4}$$

where, Y is the admittance of the structure at the plane containing the central conductor given by [11], [12].

$$Y = \epsilon_{r2}\left\{\frac{\epsilon_{r1}\coth(\beta h1) + \epsilon_{r2}\tanh(\beta h2)}{\epsilon_{r2} + \epsilon_{r1}\coth(\beta h1)\tanh(\beta h2)}\right\} + \epsilon_{r3}\left\{\frac{\epsilon_{r4}\coth(\beta h4) + \epsilon_{r3}\tanh(\beta h3)}{\epsilon_{r3} + \epsilon_{r4}\coth(\beta h3)\tanh(\beta h4)}\right\}. \tag{5}$$

The function $f(\beta)/Q$ depending upon the charge distribution can be obtained from Yamashita [10]. The four layered general structure could be easily reduced to many useful structures by changing the value of $h1$ to $h4$ and ϵ_{r1} to ϵ_{r4}. It is necessary to convert the multilayer structure into an equivalent single layer structure before using the KJ dispersion model. Verma *et al.* [12] have assumed that multilayers on the microstrip line result in modification of the dielectric constant of the substrate ϵ_{r1}. Such a modified ϵ_{r1}, called ϵ_{r1}' can be obtained from

$$\epsilon_{r1}' = \frac{(\epsilon_{eff}(0) - 1)}{q} + 1 \tag{6}$$

where q is the filling factor given by [9]

$$q = \frac{1}{2}(1 + p)$$

$$p = \left[1 + \frac{12h_1}{w}\right]^{-1/2}, \quad \text{for } \frac{w}{h_1} > 1$$

$$p = \left[1 + \frac{12h1}{w}\right]^{-1/2} + 0.04\left[1 - \frac{w}{h1}\right]^2; \quad \text{for } \frac{w}{h1} \leq 1 \tag{7}$$

Due to the variational nature of the problem the calculated value of ϵ_{r1}' is to be corrected. The correction factor can be obtained first for the ordinary microstrip line without any cover or superstrate layer and then applied to the multilayer structure. Thus, we have extended the original KJ dispersion model to the multilayer structure.

IV. Microstrip on Two Layer Substrate

The passivation layer on GaAs forms the two dielectric layer composite substrate. This structure is also used for the design of overlay capacitors and crossings. The admittance function of such structure can be obtained from (5):

$$Y = \epsilon_{r2}\left\{\frac{\epsilon_{r1} + \epsilon_{r2}\tanh(\beta h1)\tanh(\beta h2)}{\epsilon_{r1}\tanh(\beta h2) + \epsilon_{r2}\tanh(\beta h1)}\right\} + 1 \tag{8}$$

The equivalent single layer dielectric constant ϵ_{r1}' of this structure will be a function of ϵ_{r1}, ϵ_{r2}, $h1$, $h2$ and width of the conducting strip. However, a simple expression for ϵ_{r1}' could also be obtained following the analysis of Lee and Dahele [13]:

$$\epsilon_{r1}' = \frac{\epsilon_{r1}\epsilon_{r2}(h1 + h2)}{\epsilon_{r1}h2 + \epsilon_{r2}h1}. \tag{9}$$

For the thin passivation layer i.e., $h2/h1 \ll 1$, (9) reduces to ϵ_{r1}. Using the ESDT, Jansen predicts that even a very thin passivation layer of polymide ($\epsilon_{r2} = 3.5$, $h2 = 3$ μm i.e. 1.5% of the

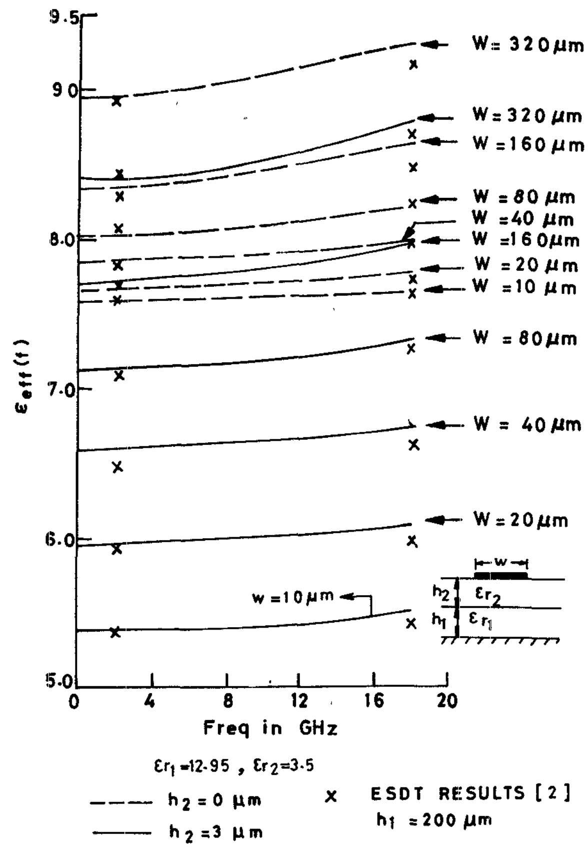

Fig. 3. Dispersion of microstrip line on two layer substrate: (---) $h_2 = 0$ μm, (—) $h_2 = 3$ μm, $h_1 = 200$ μm, ($\times$). ESDT results [2]: $\epsilon_{r1} = 12.95$, $\epsilon_{r2} = 3.5$.

TABLE I
COMPARISON OF METHODS TO REDUCE DOUBLE LAYER SUBSTRATE INTO AN EQUIVALENT SINGLE LAYER

Equivalent Single Substrate	% rms Errors 2 GHz	% rms Errors 18 GHz	% Min. Errors 2 GHz	% Min. Errors 18 GHz	% Max. Errors 2 GHz	% Max. Errors 18 GHz
$h = 200$ μm $\epsilon'_{r1} = 12.95$	0.40	0.81	0.26	0.66	1.64	3.59
$h = 203$ μm $\epsilon'_{r1} = 12.453$	0.39	0.65	0.23	0.07	1.61	2.97
$h = 203$ μm ϵ'_{r1} changes with width of strip	0.38	0.48	0.22	0.12	1.59	2.50

substrate) on GaAs ($\epsilon_{r1} = 12.95$, $h_1 = 200$ μm) reduces ϵ_{eff} by about 30%. This effect is more pronounced for the narrow strip conductor. Fig. 3 shows the dispersive behavior of microstrip line on GaAs with and without passivation. At 2 GHz the result follows closely the ESDT results of Jansen [2]. Table I compares three methods for the reduction of two layers substrate to a single equivalent layer substrate. The single layer reduction method using (6) and (7) provides an rms error less than 0.5% between 2 GHz and 18 GHz. The maximum error is towards the wide strip width at 18 GHz. However, for such a case error is 1.5% even without any passivation layer which uses only KJ dispersion model for computation. The layer reduction based upon (9) is simple but it does not show dependence of ϵ'_{r1} on the width of the conductor. However, this expression provides an rms error 0.65% which is not high. Some error has been introduced in reading the numerical values from the graphical results of the ESDT analysis.

V. SHIELDED MICROSTRIP LINE

The shielded microstrip line can be obtained from Fig. 1, whose admittance function is given by

$$Y = \epsilon_{r1} \coth(\beta h1) + \coth(\beta h3) \qquad (10)$$

Dispersion results have been obtained for $\epsilon_{r1} = 2.65, 8.875$, $h1 = 1.27$ mm, $w = 1.27$ mm and $h2/h1 = 1, 3, 9, 12$. The calculated results compared against the SDA calculation of Itoh and

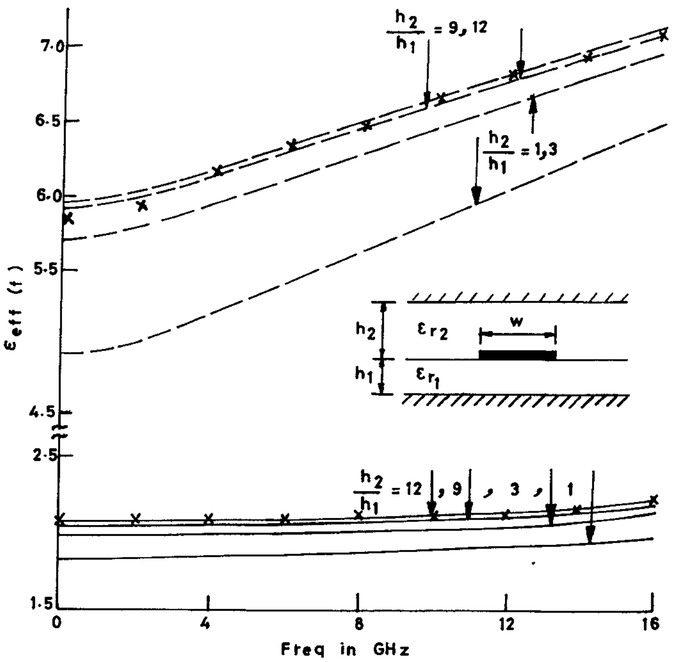

Fig. 4. Dispersion of shielded microstrip line: (---) ϵ_{r1} = 8.875, (—) ϵ_{r1} = 2.65, (×). SDA results [1]: W = 1.27 mm, h_1 = 1.27 mm.

Mitra [4] for $h2/h1 = 9$ is shown in Fig. 4. Results from the unified dispersion model follow closely the predicated behavior of the dispersion from the spectral domain analysis. The rms error of the present method compared to SDA is 0.15% and 0.45% for ϵ_{r1} = 2.65 and ϵ_{r1} = 8.875, respectively. Fig. 4 also shows the effect of a shield on $\epsilon_{\text{eff}}(f)$. This is more pronounced on the high dielectric constant substrate. The close proximity of a shield brings down $\epsilon_{\text{eff}}(f)$. This effect can be ignored for the high dielectric constant substrate for $h2/h1 > 12$ and for low dielectric substrate effect can be ignored for $h2/h1 > 3$. Some improvement in results follows if we account for the effect of the shield on the dielectric constant of the substrate and reduce the shielded structure to a single microstrip line as discussed in the Section III of this paper. However, for a shield placed far away such a modification could be ignored.

VI. Dielectric Covered Microstrip Line

Dielectric cover has been used as protective layers for lines and patches. Such layers are also needed for the design of crossings in MMIC. The effect of a cover on dispersion should be accounted for correctly when making an analysis of fast pulses on the covered microstrip line. This analysis has been carried out by Jackson and Alexopoulos [3] for Teflon substrates (ϵ_{r1} = 2.1) with GaAs superstate (ϵ_{r2} = 12.5). The admittance function of dielectric covered microstrip line follows from (5).

$$Y = \epsilon_{r1} \coth(\beta h1) + \epsilon_{r3}\left\{\frac{\epsilon_{r3} + \coth(\beta h3)}{1 + \epsilon_{r3}\coth(\beta h3)}\right\}. \tag{11}$$

For $w/h1 = 1$, the dispersion result for this structure including uncovered microstrip line and lines with superstrate (cover) of various thickness is shown in Fig. 5. Results obtained by Jackson and Alexopoulos using the method of moments are also shown for comparison. At the higher frequencies and for $h2/h1 = 0.5$ Jackson

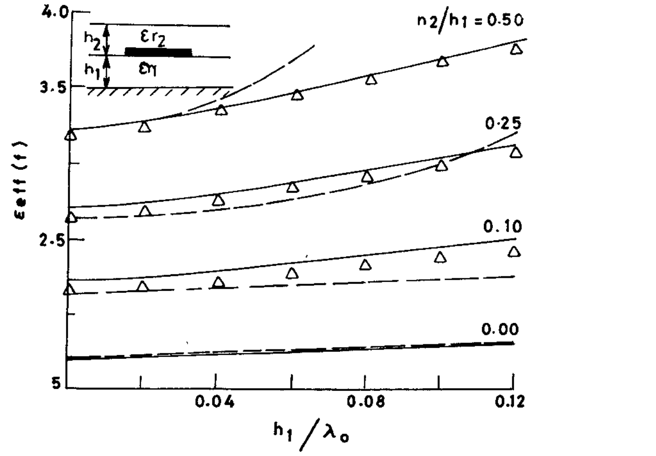

Fig. 5. Dispersion of dielectric covered microstrip line: (—) present method, (---) method of moments [3], (△) $\epsilon_{\text{eff}}(0)$ from method of moments [3], ϵ_{r1} = 2.1, ϵ_{r2} = 12.5, w/h = 1.

and Alexopoulos result departs from the present model as the confinement of power starts shifting from substrate to superstrate due to $\epsilon_{r2} \gg \epsilon_{r1}$. The present model does not account for the inversion of power that flows from substrate to superstrate. Such a case does not arise when $\epsilon_{r1} \gg \epsilon_{r2}$. Also, there is departure in $\epsilon_{\text{eff}}(0)$ calculated by the method of moments and the Fourier transformed variational method adopted in the present model. If we accept $\epsilon_{\text{eff}}(0)$ from the method of moments our dispersion result follows more closely the dispersion behavior predicted by Jackson and Alexopoulos. Moreover, the accuracy of the method of moments itself depends upon the selection of basis functions and the size of the matrix. Normally, accuracy suffers with improved computational efficiency.

VII. Conclusion

The unified dispersion model is suitable for normal microstrip lines, multilayer microstrip lines and shielded microstrip lines. This model could be extended to cover suspended structures also. For the sake of brevity such results are not included in this paper.

The numerical calculation using the unified dispersion model is very fast even on a desktop computer. Compared with the results obtained from a full-wave analysis, the computed dispersion results by this model are within 1% in the frequency range 2 GHz to 18 GHz. The results obtained by using the unified model deviates from the results of full-wave analysis when the permittivity of dielectric cover is very much higher than the permittivity of the substrate. The proposed model could be of use in CAD for design of MIC's, MMIC's and printed antennas. The unified dispersion model can be combined with any other closed form dispersion model of microstrip line.

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Channel Expansion and Tolerance Analysis of Waveguide Manifold Multiplexers*

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***Abstract*—A computer aided optimization procedure is introduced to enable the addition of extra channels to an already existing waveguide manifold multiplexer, without changing any of the existing multiplexer elements. The process provides the important advantage of the ability to expand the number of channels as required, a property which was only feasible before for channel dropping type multiplexers. The process is illustrated by practical examples that show its validity. Analysis of the effect of mechanical tolerances on the multiplexer performance is also presented to provide guide lines for the tolerance ranges in manifold multiplexer fabrication.**

I. Introduction

Waveguide manifold multiplexers have been widely used in communication satellite applications requiring high quality, low loss, high power handling capability, small size and mass [1]–[6]. Their applications cover bands from S [7], Ku [8] to millimeter wave [9] frequencies. Due to the structure complexity, experimental adjustment and optimization of initial designs was always needed to obtain satisfactory performance [1]–[5]. Recently in reference [7], precise computer modeling techniques of waveguide T-junctions and filters have been developed to a degree that the design and construction are allowed to practically achieve the final desired response without any adjustments.

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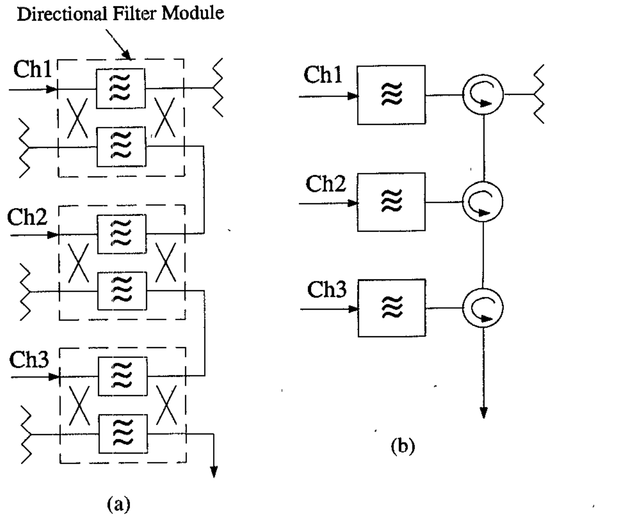

Fig. 1. Channel dropping multiplexers using directional filters. (a) Two filters and two hybrids per module. (b) One filter and one circulator per module.

Despite their desirable characteristics, waveguide manifold multiplexers have not been used in applications where the flexibility of adding channels to an existing multiplexer is required, such as in satellite earth stations, S-band TV distribution systems and cellular radio base stations. These applications have been typically served using channel dropping techniques that allow the simple cascading of "modules." Each module typically requires directional filters consisting of the equivalent of a pair of band pass filters and a pair of hybrids for each channel, or a single filter and circulator per channel, as shown in Fig. 1. Although these techniques have the advantages of eliminating interactions among channels, their performance is generally inferior to well designed waveguide manifold type multiplexers, (e.g. larger in band insertion loss, gain slope and group delay variations).

The objective of this paper is to present a computer-aided design procedure that enables the simple expansion of the number of channels of an already existing manifold multiplexer, without changing any of the elements of the existing multiplexer. Unlike the procedure described in [12], in the present paper, the original multiplexer parameters are all fixed and not allowed to change. As a result, it is now possible to expand an already deployed multiplexer in the field by adding to it new properly designed modules.

II. Modeling and Optimization

A manifold multiplexer, shown in Fig. 2, is a combination of several separated devices (T-junctions and filters) and the connecting pieces of waveguide. The multiplexer model can be built up by modeling each device, separately, to determine its scattering parameters, and then combining the scattering matrices together to obtain the scattering parameters of the $n + 1$-port multiplexer.